CUMCM-2021 Problem A

Deforming the active reflector of FAST

FAST (Five-hundred-meter Aperture Spherical radio Telescope) is the largest single-aperture radio telescope with independent intellectual property rights and the highest sensitivity in the world. It is important for China to achieve breakthroughs in key science fields and accelerate innovation-driven development.

FAST consists of an active reflector, a signal reception system (feed cabin) and associated control, measure and support systems (see Figure 1). The active reflector is an adjustable sphere consisting of several main components such as main cable net, reflection panels, down tied cables, actuators, and supporting structures. The main cable net is composed of flexible steel cables in the form of geodesic triangular grid, which is used to support the reflection panels (including the back frame). Each reflection panel is installed on one triangular grid. The whole cable net is fixed on the surrounding supporting structures. Each main cable node on the main cable net is connected with a down tied cable, and the lower endpoint of the down tied cable is connected with an actuator fixed on the ground to achieve the deformation of the main cable net. There are gaps between the reflection panels, which can ensure that the reflection panels are not crushed and pulled when the surface is deformed. The structure of the cable net, reflection panels and their connection relationships are shown in Figure 2 and Figure 3.

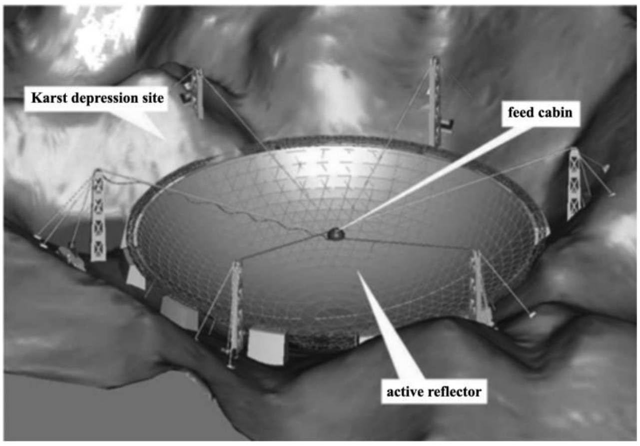


Figure 1 3-D model of FAST

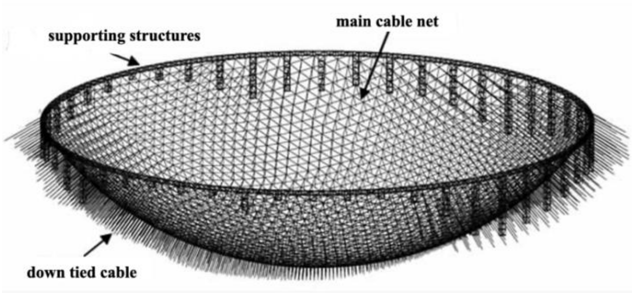
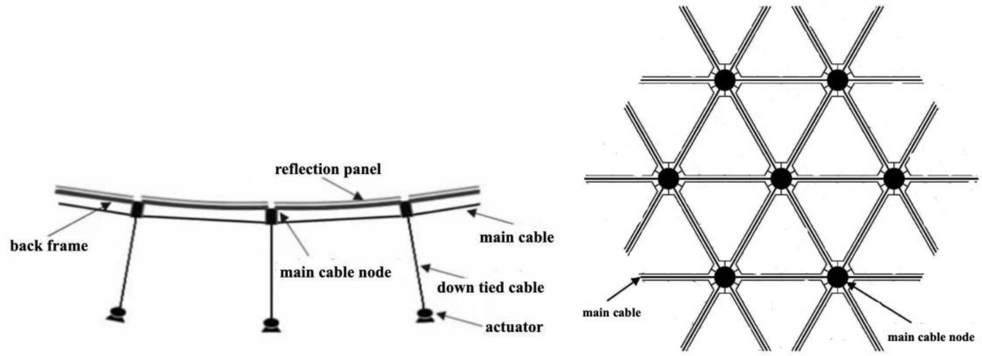


Figure 2 Main cable net and supporting structures



(a) (b)

Figure 3 The structure of the cable net, reflection panels and their connection relationships

The active reflector has two states: base state and working state. In the base state, the reflector is a sphere surface with a radius of about 300 m and an aperture of 500 m (called reference sphere). In the working state, the reflector is deformed as an approximate rotating paraboloid with an aperture of 300 m (called working paraboloid). Figure 4 shows the FAST cross-section during observation. Point C is the center of the reference sphere. The center of the receiving plane of the feed cabin can only move on the sphere (focal surface) concentric with the reference sphere, and the radius difference between the two concentric spheres is F=0.466R (R is the radius of the reference sphere, and F/R is called the focal ratio). The effective area of the signal received by the feed cabin is a central disk with a diameter of 1 m. When FAST observes one target object S along a certain direction, the center of the receiving plane of the feed cabin is moved to point P, which is the intersection of the straight line SC and the focal surface. Some reflection panels on the reference sphere are adjusted to form an approximate rotating paraboloid with the straight line SC as the symmetrical axis and P as the focal point, so that the parallel electromagnetic waves from the target object are reflected to the effective area of the feed cabin.

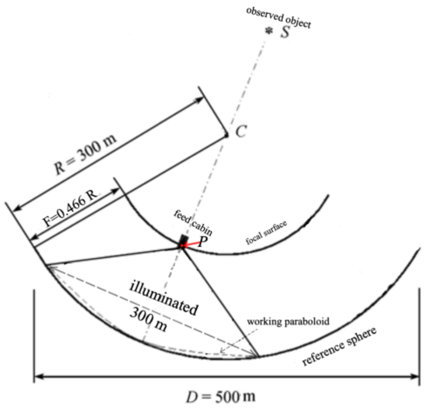


Figure 4 FAST cross-section during observation

Adjusting the reflector to a working paraboloid is the key of active reflector technology, and the process is accomplished through the cooperation of the down tied cable and the actuator. The lengths of the down tied cables are fixed. The actuators are installed radially along the reference sphere, and the bottom endpoints are fixed on the ground. The top endpoints can expand and contract radially along the reference sphere to control the down tied cables, thereby to adjust the position of the reflection panels, and finally forming the working paraboloid.

The fundamental purposes of the contest problems are: To determine an ideal paraboloid under the constraint of adjusting the reflection panels, and then adjust the reflector to the working paraboloid by adjusting the radial expansion or contraction of the actuator so that the working paraboloid is as close to the ideal paraboloid as possible and obtain the best receiving effect of electromagnetic waves reflected by the reflector.

Please establish models to solve the following problems based on the requirements and parameters in the appendix.

1. When the observed object S is located directly above the reference sphere, i.e., $\alpha = 0°, \beta = 90°$, please determine the ideal paraboloid by considering the adjustment of reflection panels.

2. Please determine the ideal paraboloid when the observed object is located at $\alpha = 36.795°, \beta = 78.169°$. Then establish a model of the reflection panel deformation by adjusting the expansion or contraction of the relevant actuators to make the reflecting surface as close to the ideal paraboloid as possible. The results should be save to the file "result.xlsx" with the fields the vertex coordinates of the ideal paraboloid, the node IDs and their coordinates, and telescopic length of each actuator for the adjusted reflecting surface within 300 m aperture. The format is shown as Annex 4.

3. Based on the reflection panel adjustment scheme in Problem 2, please give the reception ratio of the feed cabin after adjustment, i.e., the ratio of the reflected signal received in the effective area of the feed cabin receiver to the reflected signal of the reflecting surface within 300 m aperture, and compare it with the reception ratio of the reference sphere.

Appendix: Requirements and Parameters

1. There are 2226 main cable nodes in the active reflector, and 6525 main cables connecting the nodes. There are 4300 reflection panels without considering the reflection panels connected with the supporting structure. The coordinates and IDs of all main cable nodes are given in Annex 1. The coordinates of the lower endpoint of the actuator, the coordinates of the upper endpoint in the base state, and the corresponding main cable node IDs of the actuator are given in Annex 2. The corresponding main cable node IDs of the 4300 reflection panels are given in Annex 3.

2. All the main cable nodes are located on the reference sphere in the base state.

3. Each reflection panel is a part of the reference sphere. The reflection panel has many small round holes with a diameter less than 5 mm for rainwater leakage. Since the diameter of the holes is smaller than the wavelength of the observed electromagnetic waves which does not affect the reflection of waves, the panels can be regarded as non-porous.

4. The electromagnetic waves and the reflected waves are considered to propagate in a straight line.

5. The distance between adjacent nodes may vary slightly, no more than 0.07%, after the main cable nodes are adjusted.

6. The coordinates of the reflection panel vertices are regarded the same as the coordinates of the corresponding main cable nodes.

7. The actuators control the movement of the main cable node by expansion or contraction. The length of the down tied cable connecting the node to the actuator tip remains constant. The actuator expands and contracts in the radial direction of the reference sphere, and specifies it tends to the direction of the center of the sphere as the positive direction. The radial expansion of the actuator tip is assumed to be 0 in the base state. The radial expansion range is from -0.6 m to +0.6 m.

8. The orientation of the target S can be represented by azimuth α and elevation angle β (see Figure 5).

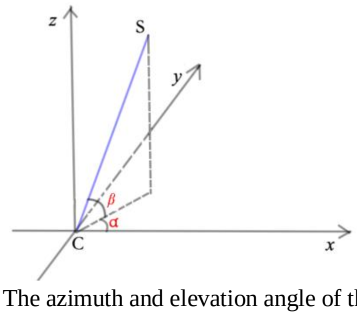


Figure 5 The azimuth and elevation angle of the target