CUMCM-2003 Problems

(Note: University-level teams can choose any one from problems A and B; College-level teams can choose any one from problems C and D)

Problem A: Transmission of SARS

SARS (Severe Acute Respiratory Syndrome) is the first epidemic that has been widely transmitted over the world since the 21st century. The outbreak and spread of SARS have brought a great influence on the economic development and people's life in China. From SARS we learn a lot of significant experience and lessons, and recognize the very importance of investigating quantitatively the transmission rule of infectious diseases to create conditions for forecast and control of the spread of infectious diseases.

Your team is asked to complete the following tasks, which are related to the mathematical modeling of SARS transmission:

(1) Evaluate the rationality and practicality for an earlier model provided in Appendix 1.

(2) Establish your own model and explain why it is better than the model in Appendix 1; in particular explain how can you establish a real predictable model that can provide enough reliable information for preventing and control the spread of SARS, and what is the difficulty to do so? Evaluate the means adopted by the health agency. For instance, estimate the influence of disease transmission if the strict isolation is taken 5 days in advance or after a 5-day delay. The data in Appendix 2 are just for reference.

(3) Collect data on an aspect of economy, which have been affected by SARS, and try to establish a mathematical model for forecasting its future effects. The data in Appendix 3 are just for your reference.

(4) Write a short essay in plain language to a local newspaper to explain the importance of the mathematical modeling of SARS.

Note: Appendices 1, 2, 3 are in Chinese and can be downloaded from http://mcm.edu.cn/mcm03/A2003.doc

(Problem A is proposed by Prof. Tang Yun from Tsinghua University and Prof. Zhou Yi-cang from Xi-An Jiaotong University, et al.)

Problem B: Truck Planning for an Opencast Iron Mine

Iron and steel industry is one of the basic industries for a country, and iron mines are the major raw material suppliers for the iron and steel industry. Many modern iron mines are working in an opencast way. Stones are loaded to trucks by electronic forklifts and then transported to several unloading positions for further delivery. In order to increase the economic benefit of the opencast mine production, an important task is to increase operation rate of the trucks and the forklifts.

There are many stone heaps in an opencast mine, which are generated by means of explosion. Each stone heap is called a forklift position, where the stones are classified into ore and rock according to the iron content. Usually, the stones with iron content not being lower than 25% on average are ores, and the others are rocks. The quantities of the ores and rocks, including the average iron content of the ores, at each position are known. Only one forklift can be placed at a forklift position. The average time for a forklift to load fully a truck is 5 minutes.

There are 5 unloading positions: one ore-yard and two railway ports for unloading the ores, and two rock-yards for unloading the rocks. Each unloading position has its own minimum output requirement for materials. In order to protect the mine resource and increase the economic benefit of the opencast mine production, the ores should be transported to the corresponding 3 unloading positions subject to the specified constraints for the iron content. Each of the 3 unloading positions has a requirement that the iron content should be within $29.5\% \pm 1\%$, and this constraint should be respected with a shift (8 hours) production. Assume that all the unloading positions are fixed within a shift, although unloading positions can be changed in the future. The average unloading time of the trucks is 3 minutes.

The carrying capacity for a truck is 154 tons, and the average speed of the trucks is 28 *km/h*. A running truck uses up a large amount of diesel oil (about 1 ton for each truck in a shift). A large amount of electronic power is needed to startup a truck, thus a truck startups at the beginning of a shift only. A started-up truck waiting in the line also consumes a large amount of electronic power, thus it should not wait in the line in principle. A forklift and an unloading position cannot provide service for two or more than two trucks simultaneously. The trucks are always fully loaded when they are working.

The road from a forklift position to an unloading position is a specialized bi-directional road of 60*m*, thus no traffic jams occur. The lengths of the roads are known.

A transportation plan in a shift should include the following information: How many forklifts should be used, and on which positions are they working? How many trucks should be used, and on which roads and how many times are they running? Due to the uncertainty and randomness, the loading and unloading time are not exactly known, thus an exact schedule is useless and you only need to specify the number of the trucks used and the running roads and running times of the trucks on each road. A feasible plan for this problem should satisfy the constraints that the trucks will not wait in line, and the quantity requirement at unloading positions should be fulfilled, and the iron contents for the ores must be satisfied. A good plan should consider one of the following two objectives:

1. The total transportation load (ton * km) is minimal, and at the meantime least trucks are used, thus the total transportation cost is minimal.

2. Use the available forklifts and trucks to obtain the maximal production quantity. (Please note that in order to remove the rocks as soon as possible, you should focus on maximizing the quantity for the rocks first. When the production quantity is maximized, you should provide a transportation plan to minimize the total transportation cost).

Please suggest mathematical models for the above two problems respectively. You should also need to design fast algorithms to solve your models. Give your production and transportation plan for the real situation listed below.

Real situation: An opencast mine has 10 forklift positions, 5 unloading positions, 7 forklifts and 20 trucks. The minimal production quantity for the unloading positions：ore-yard 12000 tons, railway port I 13000 tons, railway port II 13000 tons, rock yard I 19000 tons, rock yard II 13000 tons.

The loading and unloading positions are shown as in Figure 1, and the distances (km) are shown as follows (Table 1):

Table 1

	Forklift positions									
	1	2	3	4	5	6	7	8	9	10
Ore-yard	5.26	5.19	4.21	4.00	2.95	2.74	2.46	1.90	0.64	1.27
railway port Ⅰ	1.90	0.99	1.90	1.13	1.27	2.25	1.48	2.04	3.09	3.51
rock yard Ⅰ	5.89	5.61	5.61	4.56	3.51	3.65	2.46	2.46	1.06	0.57
rock yard Ⅱ	0.64	1.76	1.27	1.83	2.74	2.60	4.21	3.72	5.05	6.10
railway port Ⅱ	4.42	3.86	3.72	3.16	2.25	2.81	0.78	1.62	1.27	0.50

The quantities of the ores and rocks, including the average iron content of the ores, are already known for each of the forklift positions as following (Table 2):

Table 2

	Forklift positions									
	1	2	3	4	5	6	7	8	9	10
Ore Quantity	0. 95	1. 05	1. 00	1. 05	1. 10	1. 25	1. 05	1. 30	1. 35	1. 25
Rock quantity	1. 25	1. 10	1. 35	1. 05	1. 15	1. 35	1. 05	1. 15	1. 35	1. 25
Iron Content	30%	28%	29%	32%	31%	33%	32%	31%	33%	31%

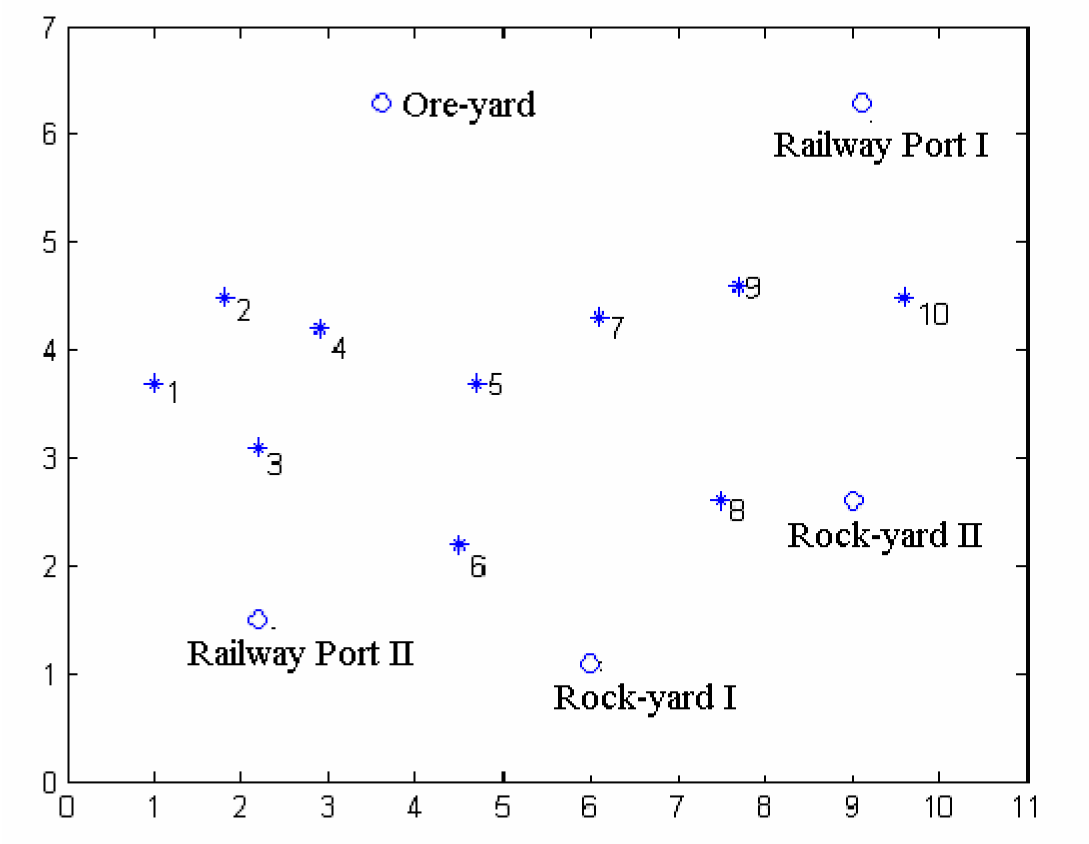


Figure 1
(“* ”represents forklift position, “o” represents unloading position)

(Problem B is proposed by Prof. Fang Shu-cheng from Jilin University, China)

Problem C:

(Same as Problem A)

Problem D: Speedily Crossing the Yangtze River

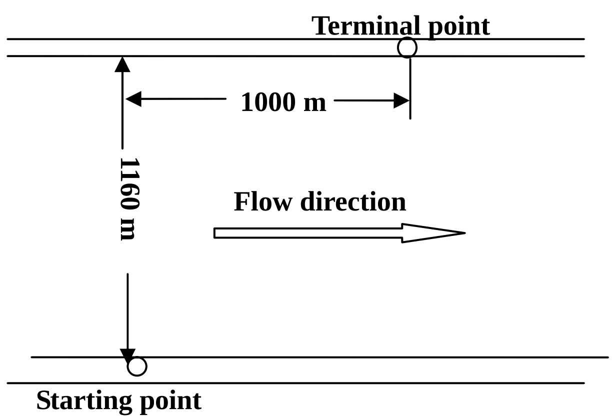
"Crossing the Yangtze River" is one of the symbols of Wuhan (the capital of Hubei province of China). In 9 September 1934, 44 swimmers from Wuhan garrison brigade and sports circles participated in an activity of crossing the Yangtze River, which was firstly held in Wuhan. The distance between the starting point at Hanyang Gate Quay in Wuchang and the terminal point at Sanbei Quay in Hankou is about 5,000 meters. There were 40 swimmers reached the terminal. And General Zhang Xueliang specially awarded to the champion winner a silver medal carved with a Chinese idiom "making vigorous effort to turn the tide".

A "Challenging race of crossing the Yangtze River" was held again in Wuhan in 2001. This race has been named "Wuhan International Challenging Race of Crossing the Yangtze River" since 2002, and will be held once a year on 1 May each year. Owing to the unpredictable regimen and weather conditions, this kind race on the public water area is more challenging and enjoyable. The starting point is at Hanyang Gate Quay in Wuchang and the terminal point is at Nan-An-Zui Quay in Hanyang. According to the report from newspapers, the average temperature of water is about 16.8°C and the average velocity of the water flow is about 1.89 m/s on 1 May 2002. There were 186 domestic and foreign swimmers participated in this race, half of them are professionals, but there were only 34 of them reached the terminal, and most of the other 152 swimmers were washed away to the lower reaches of the river because of their wrong choice of the swimming route. The record of the first place winner is 14 minutes and 8 seconds.

Suppose that two banks of the river are parallel and the distance between them is about 1,160 meters and the distance between the opposite point to the starting point at the north bank and the terminal point is about 1,000 meters (see the sketch map).

Your team is invited to analyze the situations mentioned above through mathematical modeling and answer the following questions:

1. Suppose that swimmers' velocity (speed and direction) keeps constant during the whole course and the velocity of the flow at each point of the crossing region also keeps constant. What should be the route of the first place winner and what is her speed and direction? Try to make a comment for the swimmer who can keep his or her speed at 1.5 m/s about the direction which can guarantee him or her to reach the terminal and estimate the time he or she will take.



2. Under the above assumptions, if a swimmer always swims along with the direction perpendicular to the opposite bank, could he or she reach the terminal? Why the numbers of swimmers who reached the terminal in 1934 and 2002 are so different? Try to give the conditions swimmers must fulfill which can guarantee them to reach the terminal.
3. If the velocity of the water flow is as follows (suppose that the direction perpendicular up to the south bank of the river is the positive y-axes)

$$v(y)=\begin{cases}1.47m/s, & 0\le y\le 200\ m\\ 2.11m/s, & 200<y<960\ m\\ 1.47m/s, & 960\le y\le 1160\ m\end{cases}$$

and the swimmer's speed is still 1.5 m/s, please give the direction which can guarantee him or her to reach the terminal and estimate how much time he or she spend.
4. If the velocity of the water flow is continuously distributed, for instance,

$$v(y)=\begin{cases}\frac{2.28}{200}y, & 0\le y\le 200\\ 2.28, & 200\le y\le 960\\ \frac{2.28}{200}(1160-y), & 960\le y\le 1160\end{cases}$$

or you can set your own reasonable velocity distribution, how can you deal with this situation?
5. Write a comment in plain language on what strategies swimmers might use for persons who are willing to participate in the race of crossing the Yangtze River next time.
6. Are there any other applications of your mathematical model?

(Problem D is proposed by Prof. Yin Jian-su from Central China Agriculture University and Prof. Ye Qi-xiao from Beijing Institute of Technology, et al.)